Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 2

Tuesday 14 November 2017 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 15]

A group of 800 students answered 40 questions on a category of their choice out of History, Science and Literature.

For each student the category and the number of correct answers, N, was recorded. The results obtained are represented in the following table.

		Number of correct answers, N				
		$1 \le N \le 10$	$11 \le N \le 20$	$21 \le N \le 30$	$31 \le N \le 40$	**Total number of students**
Category	History	46	80	68	39	**233**
	Science	37	82	85	56	**260**
	Literature	31	110	104	62	**307**
	Total number of students	**114**	**272**	**257**	**157**	**800**

(a) State whether N is a discrete or a continuous variable. [1]

(b) Write down, for N,

(i) the modal class;

(ii) the mid-interval value of the modal class. [2]

(c) Use your graphic display calculator to estimate

(i) the mean of N;

(ii) the standard deviation of N. [3]

A χ^2 test at the 5% significance level is carried out on the results. The critical value for this test is 12.592.

(d) Find the expected frequency of students choosing the Science category and obtaining 31 to 40 correct answers. [2]

(This question continues on the following page)

(Question 1 continued)

(e) Write down

(i) the null hypothesis for this test;

(ii) the number of degrees of freedom. [2]

(f) Write down

(i) the p-value for the test;

(ii) the χ^2 statistic. [3]

(g) State the result of the test. Give a reason for your answer. [2]

2. [Maximum mark: 15]

Rosa joins a club to prepare to run a marathon. During the first training session Rosa runs a distance of 3000 metres. Each training session she increases the distance she runs by 400 metres.

(a) Write down the distance Rosa runs

(i) in the third training session;

(ii) in the nth training session. [3]

A marathon is 42.195 kilometres.

In the kth training session Rosa will run further than a marathon for the first time.

(b) Find the value of k. [2]

(c) Calculate the total distance, in **kilometres**, Rosa runs in the first 50 training sessions. [4]

Carlos joins the club to lose weight. He runs 7500 metres during the first month.
The distance he runs increases by 20% each **month**.

(d) Find the distance Carlos runs in the fifth month of training. [3]

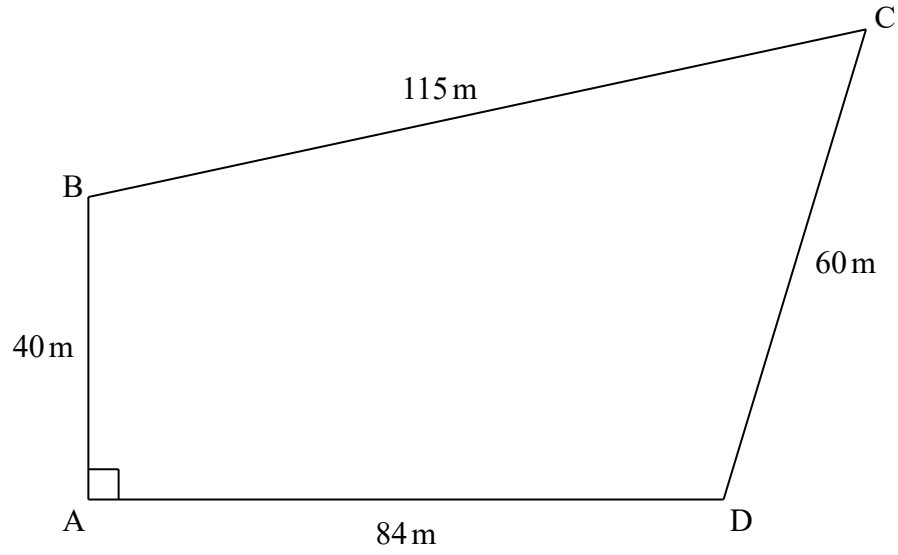
(e) Calculate the total distance Carlos runs in the first year. [3]

3. [Maximum mark: 13]

Abdallah owns a plot of land, near the river Nile, in the form of a quadrilateral $ABCD$. The lengths of the sides are $AB = 40\,\text{m}$, $BC = 115\,\text{m}$, $CD = 60\,\text{m}$, $AD = 84\,\text{m}$ and angle $B\hat{A}D = 90^\circ$.

This information is shown on the diagram.

diagram not to scale



(a) Show that $BD = 93\,\text{m}$ correct to the nearest metre. [2]

(b) Calculate angle $B\hat{C}D$. [3]

(c) Find the area of $ABCD$. [4]

The formula that the ancient Egyptians used to estimate the area of a quadrilateral $ABCD$ is

$$\text{area} = \frac{(AB + CD)(AD + BC)}{4}.$$

Abdallah uses this formula to estimate the area of his plot of land.

(d) (i) Calculate Abdallah's estimate for the area.

(ii) Find the percentage error in Abdallah's estimate. [4]

4. [Maximum mark: 17]

A company performs an experiment on the efficiency of a liquid that is used to detect a nut allergy.

A group of 60 people took part in the experiment. In this group 26 are allergic to nuts. One person from the group is chosen at random.

(a) Find the probability that this person is **not** allergic to nuts. [2]

A second person is chosen from the group.

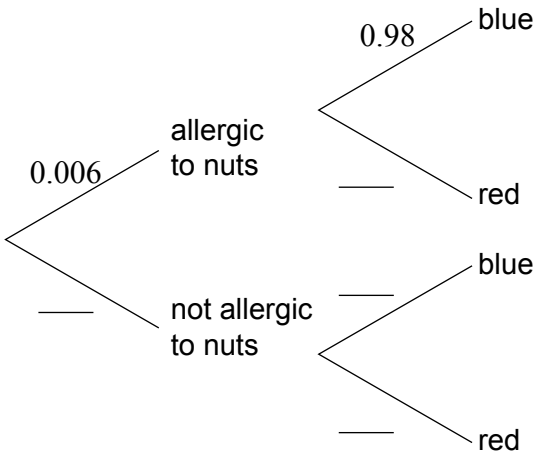
(b) Find the probability that both people chosen are **not** allergic to nuts. [2]

When the liquid is added to a person's blood sample, it is expected to turn blue if the person is allergic to nuts and to turn red if the person is not allergic to nuts.

The company claims that the probability that the test result is correct is 98% for people who are allergic to nuts and 95% for people who are not allergic to nuts.

It is known that 6 in every 1000 adults are allergic to nuts.

This information can be represented in a tree diagram.



(c) **Copy** and complete the tree diagram. [3]

An adult, who was not part of the original group of 60, is chosen at random and tested using this liquid.

(d) Find the probability that this adult is allergic to nuts and the liquid turns blue. [2]

(e) Find the probability that the liquid turns blue. [3]

(f) Find the probability that the tested adult is allergic to nuts given that the liquid turned blue. [3]

(This question continues on the following page)

(Question 4 continued)

The liquid is used in an office to identify employees who might be allergic to nuts. The liquid turned blue for $\mathbf{38}$ **employees**.

(g) Estimate the number of employees, from this 38, who are allergic to nuts. [2]

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5. [Maximum mark: 16]

A function f is given by $f(x) = (2x + 2)(5 - x^2)$.

(a) Find the **exact** value of each of the zeros of f. [3]

(b) (i) Expand the expression for $f(x)$.

(ii) Find $f'(x)$. [4]

(c) Use your answer to part (b)(ii) to find the values of x for which f is increasing. [3]

(d) **Draw** the graph of f for $-3 \leq x \leq 3$ and $-40 \leq y \leq 20$. Use a scale of $2\,\text{cm}$ to represent 1 unit on the x-axis and $1\,\text{cm}$ to represent 5 units on the y-axis. [4]

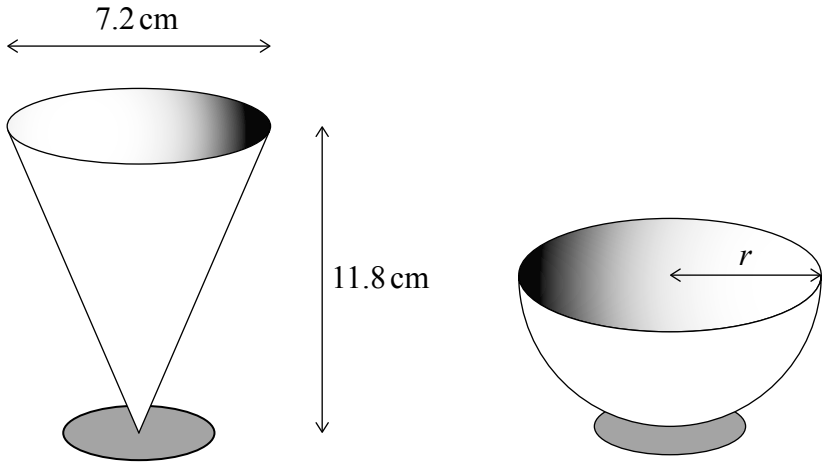
The graph of the function $g(x) = 5^x + 6x - 6$ intersects the graph of f.

(e) Write down the coordinates of the point of intersection. [2]

6. [Maximum mark: 14]

A restaurant serves desserts in glasses in the shape of a cone and in the shape of a hemisphere. The diameter of a cone shaped glass is $7.2\,\text{cm}$ and the height of the cone is $11.8\,\text{cm}$ as shown.

diagrams not to scale



(a) Show that the volume of a cone shaped glass is $160\,\text{cm}^3$, correct to 3 significant figures. [2]

The volume of a hemisphere shaped glass is $225\,\text{cm}^3$.

(b) Calculate the radius, r, of a hemisphere shaped glass. [3]

The restaurant offers two types of dessert.

The **regular dessert** is a hemisphere shaped glass completely filled with chocolate mousse. The cost, to the restaurant, of the chocolate mousse for one regular dessert is $\$1.89$.

(c) Find the cost of $100\,\text{cm}^3$ of chocolate mousse. [2]

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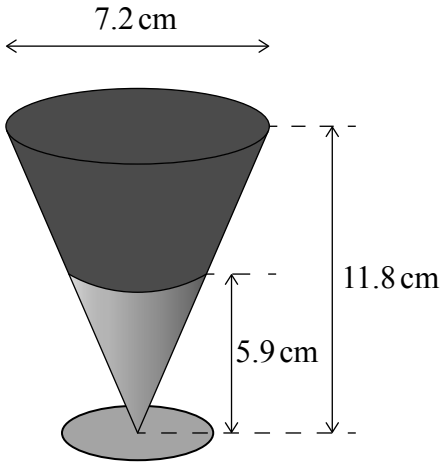
(Question 6 continued)

The **special dessert** is a cone shaped glass filled with two ingredients. It is first filled with orange paste to half of its height and then with chocolate mousse for the remaining volume.

diagram not to scale



(d) Show that there is $20\,\text{cm}^3$ of orange paste in each special dessert. [2]

The cost, to the restaurant, of $100\,\text{cm}^3$ of orange paste is $\$7.42$.

(e) Find the total cost of the ingredients of one special dessert. [2]

A chef at the restaurant prepares 50 desserts; x regular desserts and y special desserts. The cost of the ingredients for the 50 desserts is $\$111.44$.

(f) Find the value of x. [3]